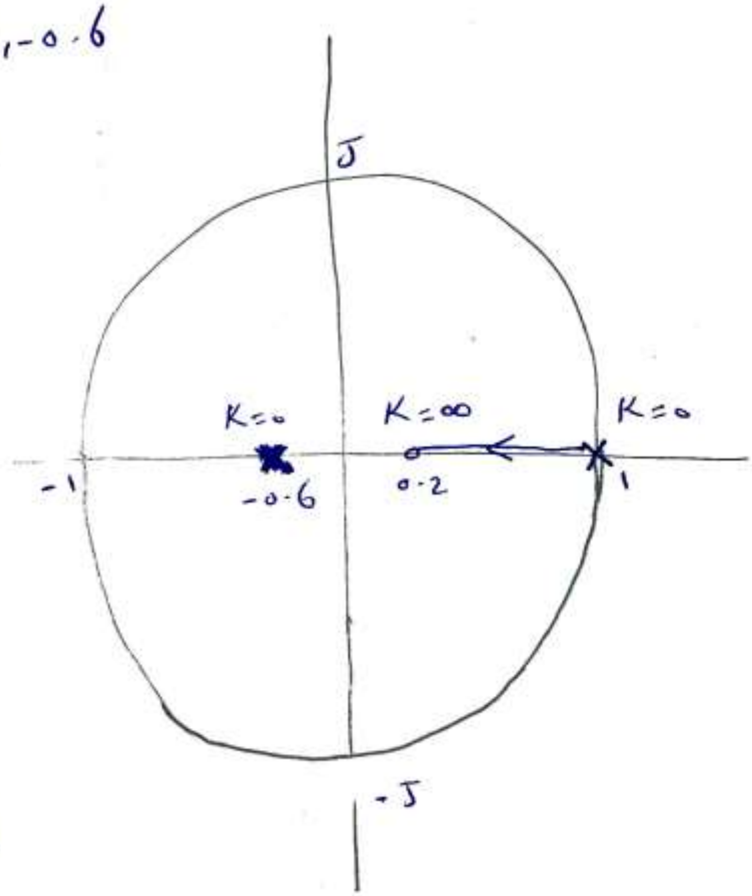
Digital Control. Lec 6

Ex: $\overline{GH}(z) = \dfrac{K(z-0.2)}{(z-1)(z+0.6)^2}$; Draw the root locus and find the range of K for stability

1) Poles : $n_p = 3 \Rightarrow 1, -0.6, -0.6$

Zeros : $n_z = 1 \Rightarrow 0.2$

2) Z - Plane



3) Real Part

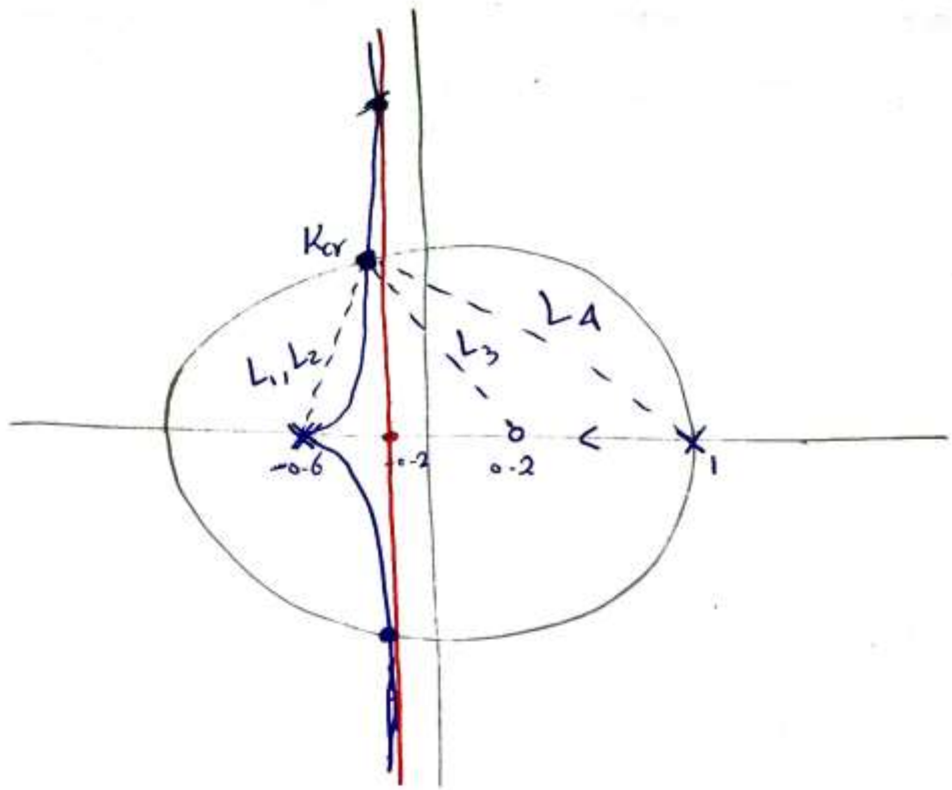
from 1 to 0.2

4) Asymptotes

no. of Asym. $= n_p - n_z = 2$

$$C_A = \frac{\sum Poles - \sum zeros}{n_p - n_z} = \frac{(1-0.6-0.6)-(0.2)}{2} = -0.2$$

$$\theta = \frac{(2L+1)180}{n_p - n_z} = \begin{cases} \theta_1 = 90 \\ \theta_2 = -90 \end{cases}$$



$\boxed{5}$ Determine K_{cr}:

$$K_{cr} = \frac{\prod \text{Poles}}{\prod \text{zeros}} = \frac{\underbrace{L_1 * L_2}_{L_1^2} * L_4}{L_3}$$

another solution $\Rightarrow$ use bilinear transformation

$$Z = \frac{1+r}{1-r}$$

ch. equation: $1 + \overline{GH(z)} = 0$

$$1 + \frac{K(Z-0.2)}{(Z-1)(Z-0.6)} = 0$$

$$(z-1)(z-0.6) + K(z-0.2) = 0$$

$$Z = \frac{1+r}{1-r}$$

$\cancel{(0.32+1.2K)}$

$$(0.32+1.2K)r^3+(2.56-1.6K)r^2+(5.12-0.4K)r$$
$$+0.8K=0$$

$$\begin{array}{c|ll} r^3 & \underline{\underline{0.32+1.2K>0}} & 5.12-0.4K \\ r^2 & \underline{\underline{2.56-1.6K>0}} & 0.8K \\ r^1 & \underline{\underline{A>0}} & \\ r^0 & \underline{\underline{0.8K>0}} & \end{array}$$

الشروط اللى تحتها خطين يكون
تقاطعها هو range of K

1) $0.32+1.2K>0 \Rightarrow K>-0.267$

2) $2.56-1.6K>0 \Rightarrow \cancel{K} \; K<1.6$

3) $0.8K>0 \Rightarrow K>0$

4) $A>0$

$$\left[(2.56-1.6K)(5.12-0.4K)\right]-0.8K(0.32+1.2K)>0$$

$$-0.32K^2-9.472K+13.107>0$$

$$K^2+29.6K-40.96<0 \qquad K_{1,2}=1.32 \;\&\; -30.92$$

$$(K-1.32)(K+30.92)<0$$

قوسين أقل من الصفر اما أحدهما موجب والآخر سالب أو العكس.

$$(K-1.32)(K+30.92) < 0$$

$$\overset{+}{K > 1.32} \qquad \overset{-}{K < -30.92} \Rightarrow \times$$

$$\boxed{\overset{-}{K < 1.32} \qquad \overset{+}{K > -30.92}} \checkmark \quad (4)$$

The range of K for stability

$$\boxed{0 < K < 1.32}$$

Bode diagram

→ To study system Properties in freq. domain for a discrete time system, we use bilinear Transformation, to get the system in cont. time domain.

bilinear Transformation $\Rightarrow \boxed{Z = \frac{1+r}{1-r}}$

* Bode diagram is one of relative stability methods.

→ given open loop Digital T.F $\overline{GH}(z)$,

To Draw the bode diagram.

1) Map from Z-domain to r-domain.

$$Z = \frac{1+r}{1-r}$$

$$\overline{GH}(z) \longrightarrow \overline{GH}(r)$$

2) $r \longrightarrow J\omega_r$

$$\overline{GH}(r) \longrightarrow \overline{GH}(J\omega_r)$$

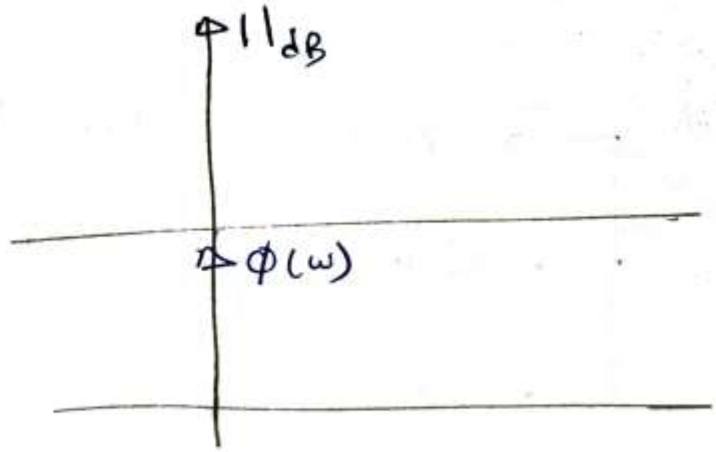
3) $$|\overline{GH}(J\omega_r)| = \frac{|\text{البسط}|}{|\text{المقام}|}$$

$$|\overline{GH}(J\omega_r)|_{dB} = 20 \log |\overline{GH}(J\omega_r)|$$

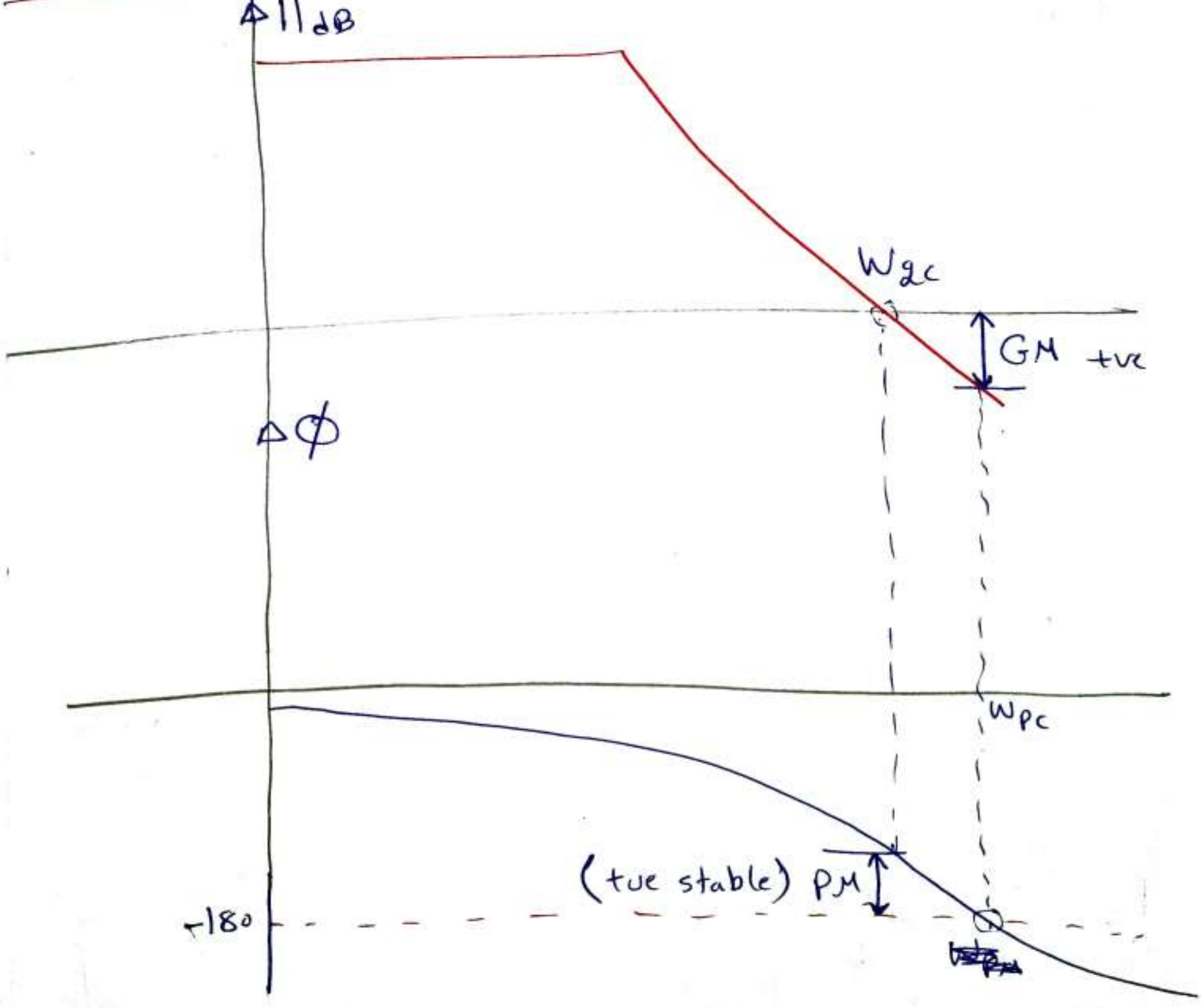
4) $$\phi(\omega_r) = \angle \text{بسط} - \angle \text{المقام}$$

$$= \tan^{-1}\left(\frac{\text{Imag.}}{\text{real}}\right)$$

⑤ Draw

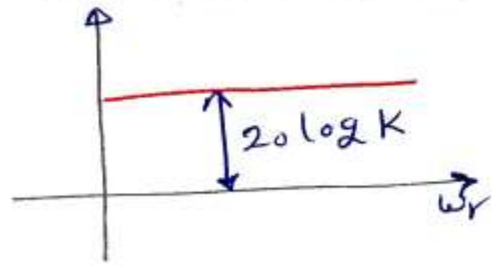
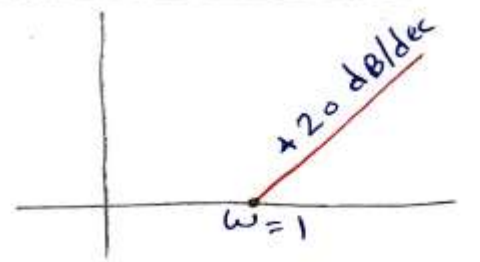
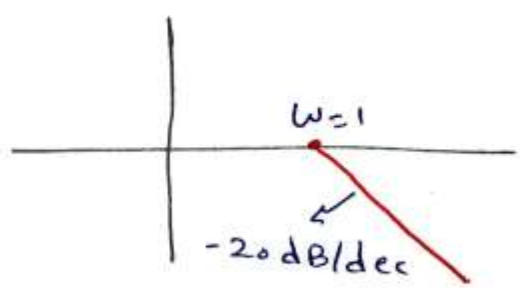
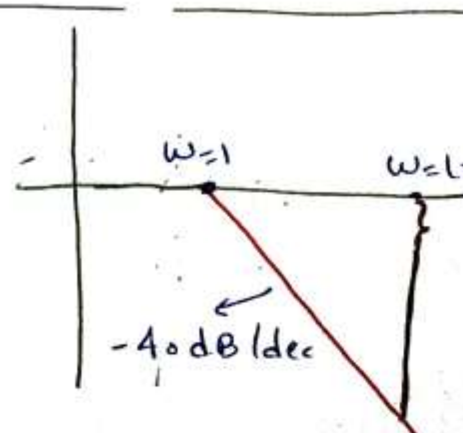


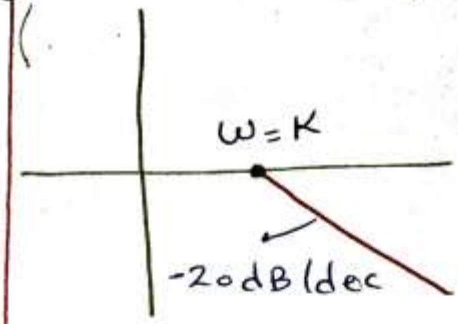
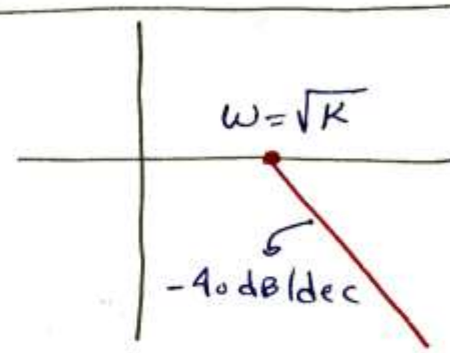
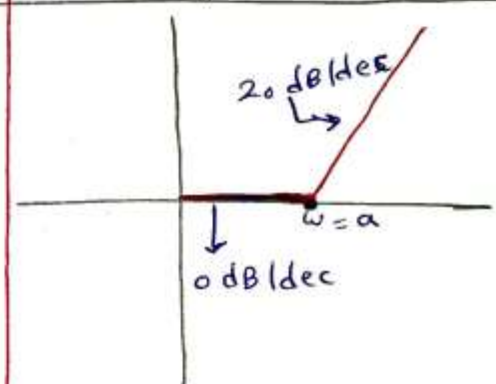
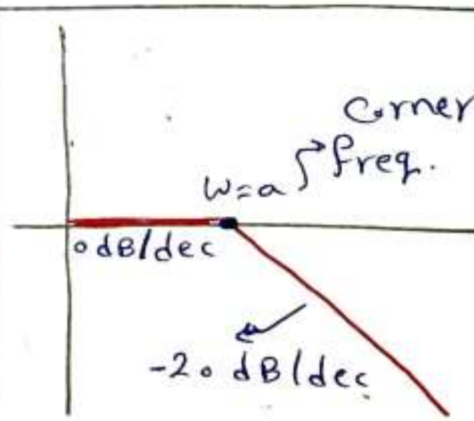
f-r example



GM
- → > 0 (stable)
- → < 0 (unstable)
- → $= 0$ (critically stable)

Common terms

Term	$\phi(\omega_r)$	$\lvert\ \rvert_{dB}$
K	—	
$s \to r \to j\omega_r$	$+90$	
$\frac{1}{s} \to \frac{1}{r} \to \frac{1}{j\omega_r}$	-90	
$\frac{1}{s^2} \to \frac{1}{r^2} \to \frac{1}{j\omega_r} \cdot \frac{1}{j\omega_r}$	-180	

$\frac{K}{s} \to \frac{K}{r} \to \frac{K}{J\omega_r}$	-90	
$\frac{K}{s^2} \longrightarrow \frac{K}{r^2}$	-180	
$\left(1+\frac{s}{a}\right) \to 1+\frac{r}{a}$ $\hookrightarrow 1+J\frac{\omega_r}{a}$	$\tan^{-1}\left(\frac{\omega_r}{a}\right)$	
$\frac{1}{1+\frac{s}{a}} \to \frac{1}{1+\frac{r}{a}}$ $\frac{1}{1+J\frac{\omega_r}{a}}$	$-\tan^{-1}\left(\frac{\omega_r}{a}\right)$	

$\boxed{EX\ 1}$ Draw Bode diagram for the following system:-

$$\overline{GH}(z) = \frac{0.5\,(z+0.76)}{(z-1)(z-0.45)}$$

and find W_{gc} & W_{pc} & GM & PM

1) $$Z = \frac{1+r}{1-r}$$

$$GH(r) = \frac{0.5\left(\frac{1+r}{1-r}+0.76\right)}{\left(\frac{1+r}{1-r}-1\right)\left(\frac{1+r}{1-r}-0.45\right)}$$

$$= \frac{0.5(1-r)\left(1+r+0.76(1-r)\right)}{\left(1+r-1(1-r)\right)\left(1+r-0.45(1-r)\right)}$$

$$= \frac{0.5\,(1-r)(1.76+0.24r)}{2r\,(0.55+1.45\,r)}$$

$$GH(r) = \frac{0.5\,(1-r)*1.76\left(1+\frac{r}{7.33}\right)}{2(0.5)\,r\left(1+\frac{r}{0.38}\right)}$$

$$\overline{GH}(r) = \frac{0.8\,(1-r)\left(1+\frac{r}{7.33}\right)}{r\left(1+\frac{r}{0.38}\right)}$$

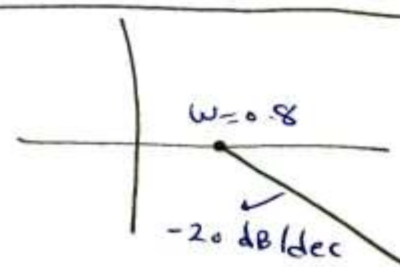
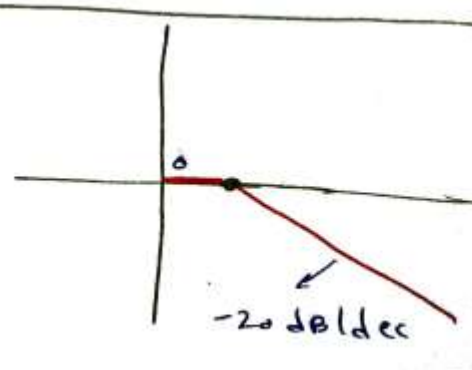
$$r \longrightarrow J\omega_r$$

$$GH(J\omega_r) = \frac{0.8}{J\omega_r}\cdot\frac{(1-J\omega_r)\left(1+J\frac{\omega_r}{7.33}\right)}{\left(1+J\frac{\omega_r}{0.38}\right)}$$

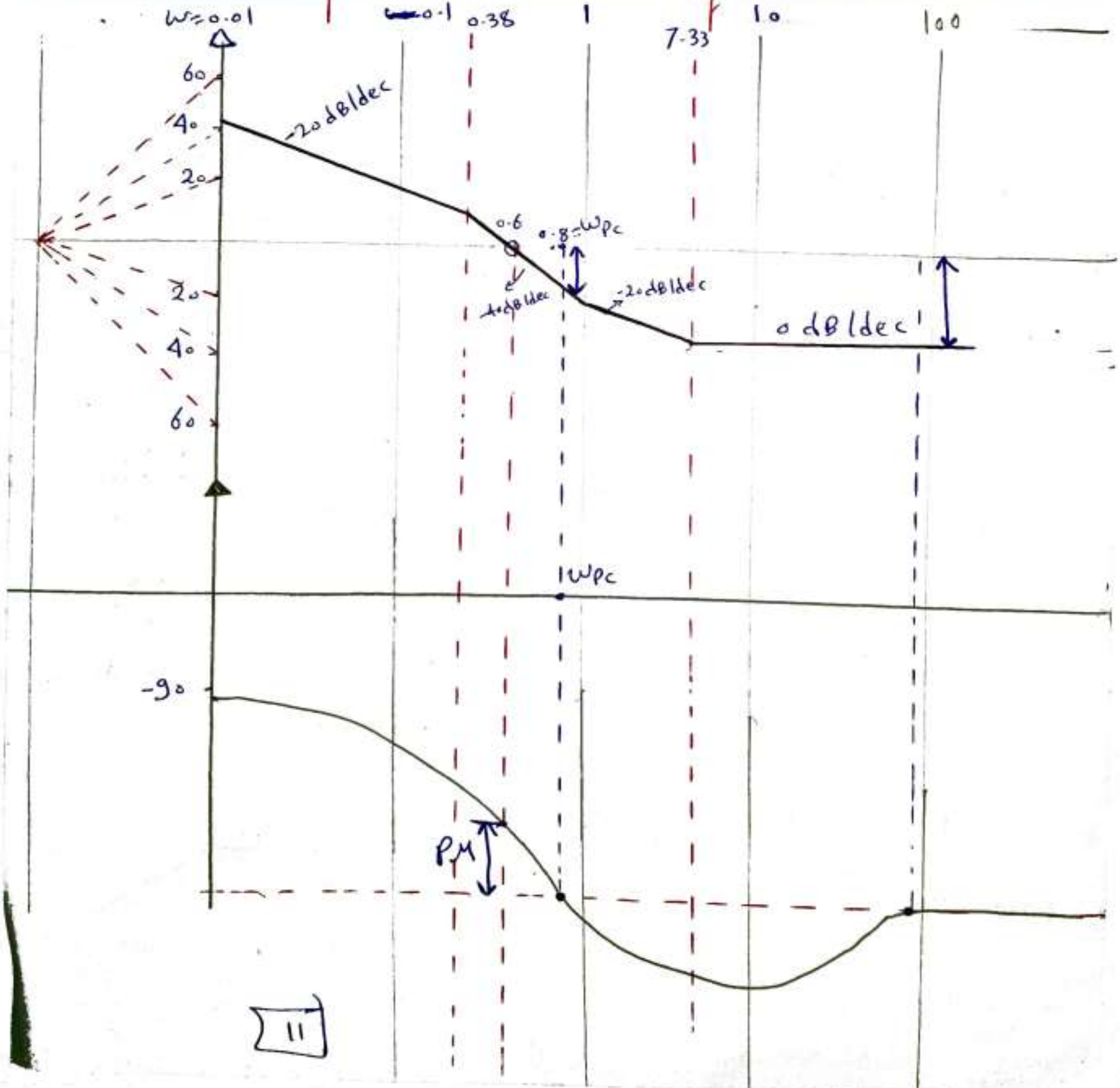
$$\phi(\omega_r) = -\tan^{-1}(\omega_r) + \tan^{-1}\left(\frac{\omega_r}{7.33}\right) - 90 - \tan^{-1}\left(\frac{\omega_r}{0.38}\right)$$

$$\omega_r = 0 \implies \phi(\omega_r) = -90$$

$$\omega_r = \infty \implies \phi(\omega_r) = -180^\circ$$

Term	$\phi(\omega_r)$	$\lvert\ \rvert_{dB}$
$\frac{0.8}{r} \to \frac{0.8}{J\omega_r}$	-90	
$\frac{1}{1+J\frac{\omega_r}{0.38}}$	$-\tan^{-1}\left(\frac{\omega_r}{0.38}\right)$	

Term	$\phi(\omega)$	$\| \|_{dB}$
$1-J\frac{\omega_r}{1}$	$-\tan^{-1}(\omega_r)$	0 dB/dec, +20 dB/dec, $\omega=1$
$1+J\frac{\omega_r}{7.33}$	$\tan^{-1}\frac{\omega_r}{7.33}$	0 dB/dec, +20 dB/dec, $\omega=7.33$



ω_r	0	0.01	0.1	0.38	0.6	1
ϕ	-90	-92°	-109.67	-152.8	-173.94	-196.43°

ω_r	7.33	100	∞
ϕ	-209.47	-183.4°	-180

← لو عندك قيمتين لل GM تأخذ أسوأهم.

← يعني القيمة الأصغر.